Conversion to N/m^2 (Pa)	Conversion from atm
1.0 atm $= 1.013 \times 10^5$ N/m^2	1.0 atm $= 1.013 \times 10^5$ N/m^2
1.0 dyne/cm^2 $= 0.10$ N/m^2	1.0 atm $= 1.013 \times 10^6$ dyne/cm^2
1.0 kg/cm^2 $= 9.8 \times 10^4$ N/m^2	1.0 atm $= 1.013$ kg/cm^2
1.0 lb/in.^{2} $= 6.90 \times 10^3$ N/m^2	1.0 atm $= 14.7$ lb/in.^{2}
1.0 mm Hg $= 133$ N/m^2	1.0 atm $= 760$ mm Hg
1.0 cm Hg $= 1.33 \times 10^3$ N/m^2	1.0 atm $= 76.0$ cm Hg
1.0 cm water $= 98.1$ N/m^2	1.0 atm $= 1.03 \times 10^3$ cm water
1.0 bar $= 1.000 \times 10^5$ N/m^2	1.0 atm $= 1.013$ bar
1.0 millibar $= 1.000 \times 10^2$ N/m^2	1.0 atm $= 1013$ millibar

Table 11.2 Conversion Factors for Various Pressure Units

11.7 Archimedes' Principle

When you rise from lounging in a warm bath, your arms feel strangely heavy. This is because you no longer have the buoyant support of the water. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected? (See Figure 11.18.)
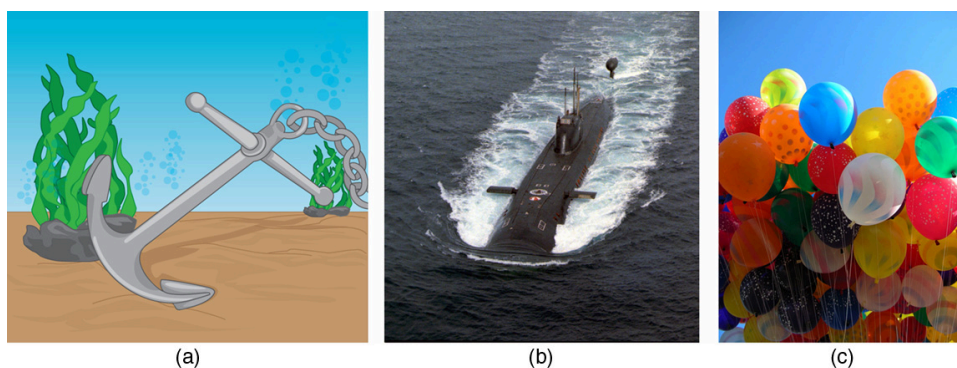

Figure 11.18 (a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (credit: Allied Navy) (c) Helium-filled balloons tug upward on their strings, demonstrating air's buoyant effect. (credit: Crystl)

Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on the top of the object. There is a net upward, or **buoyant force** on any object in any fluid. (See Figure 11.19.) If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.
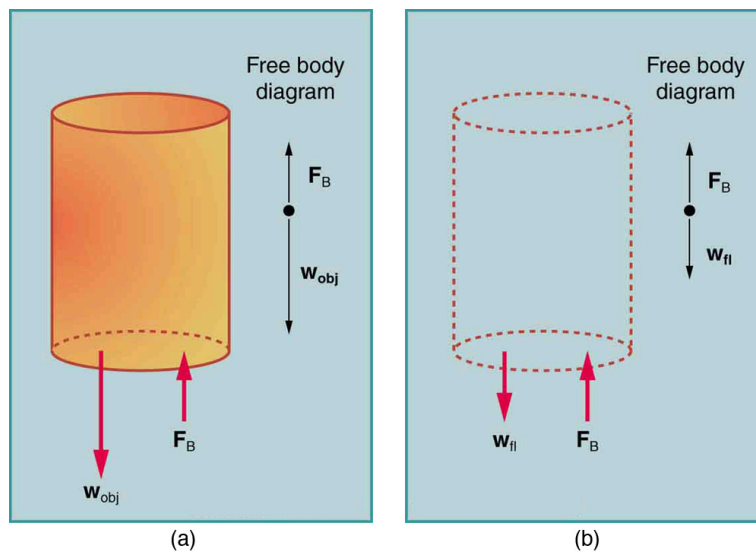
Buoyant Force

The buoyant force is the net upward force on any object in any fluid.
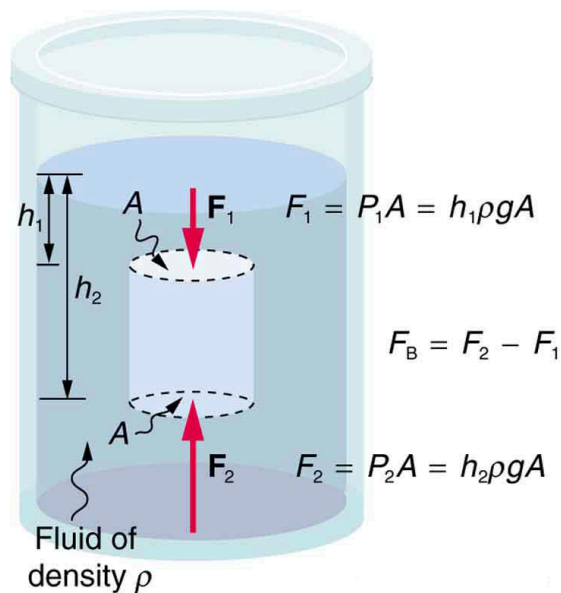
Figure 11.19 Pressure due to the weight of a fluid increases with depth since $P = h\rho g$. This pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder. Their difference is the buoyant force $\mathbf{F}_B$. (Horizontal forces cancel.)

Just how great is this buoyant force? To answer this question, think about what happens when a submerged object is removed from a fluid, as in Figure 11.20.

Figure 11.20 (a) An object submerged in a fluid experiences a buoyant force F_B. If F_B is greater than the weight of the object, the object will rise. If F_B is less than the weight of the object, the object will sink. (b) If the object is removed, it is replaced by fluid having weight w_{fl}. Since this weight is supported by surrounding fluid, the buoyant force must equal the weight of the fluid displaced. That is, $F_B = w_{fl}$, a statement of Archimedes' principle.

The space it occupied is filled by fluid having a weight w_{fl}. This weight is supported by the surrounding fluid, and so the buoyant force must equal w_{fl}, the weight of the fluid displaced by the object. It is a tribute to the genius of the Greek mathematician and

inventor Archimedes (ca. 287–212 B.C.) that he stated this principle long before concepts of force were well established. Stated in words, **Archimedes' principle** is as follows: The buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$F_B = w_{fl},$$
11.30

where F_B is the buoyant force and w_{fl} is the weight of the fluid displaced by the object. Archimedes' principle is valid in general, for any object in any fluid, whether partially or totally submerged.

Archimedes' Principle

According to this principle the buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$F_B = w_{fl},$$
11.31

where F_B is the buoyant force and w_{fl} is the weight of the fluid displaced by the object.

Humm ... High-tech body swimsuits were introduced in 2008 in preparation for the Beijing Olympics. One concern (and international rule) was that these suits should not provide any buoyancy advantage. How do you think that this rule could be verified?

Making Connections: Take-Home Investigation

The density of aluminum foil is 2.7 times the density of water. Take a piece of foil, roll it up into a ball and drop it into water. Does it sink? Why or why not? Can you make it sink?

Floating and Sinking

Drop a lump of clay in water. It will sink. Then mold the lump of clay into the shape of a boat, and it will float. Because of its shape, the boat displaces more water than the lump and experiences a greater buoyant force. The same is true of steel ships.

✳ EXAMPLE 11.8

Calculating buoyant force: dependency on shape

(a) Calculate the buoyant force on 10,000 metric tons $(1.00 \times 10^7 \text{ kg})$ of solid steel completely submerged in water, and compare this with the steel's weight. (b) What is the maximum buoyant force that water could exert on this same steel if it were shaped into a boat that could displace $1.00 \times 10^5 \text{ m}^3$ of water?

Strategy for (a)

To find the buoyant force, we must find the weight of water displaced. We can do this by using the densities of water and steel given in Table 11.1. We note that, since the steel is completely submerged, its volume and the water's volume are the same. Once we know the volume of water, we can find its mass and weight.

Solution for (a)

First, we use the definition of density $\rho = \frac{m}{V}$ to find the steel's volume, and then we substitute values for mass and density. This gives

$$V_{st} = \frac{m_{st}}{\rho_{st}} = \frac{1.00 \times 10^7 \text{ kg}}{7.8 \times 10^3 \text{ kg/m}^3} = 1.28 \times 10^3 \text{ m}^3.$$
11.32

Because the steel is completely submerged, this is also the volume of water displaced, V_w. We can now find the mass of water displaced from the relationship between its volume and density, both of which are known. This gives

$$m_{\mathrm{w}} = \rho_{\mathrm{w}} V_{\mathrm{w}} = (1.000 \times 10^3 \ \mathrm{kg/m^3})(1.28 \times 10^3 \ \mathrm{m^3})$$
$$= 1.28 \times 10^6 \ \mathrm{kg}.$$

<div style="text-align:right">11.33</div>

By Archimedes' principle, the weight of water displaced is $m_{\mathrm{w}} g$, so the buoyant force is

$$F_{\mathrm{B}} = w_{\mathrm{w}} = m_{\mathrm{w}} g = \left(1.28 \times 10^6 \ \mathrm{kg}\right)\left(9.80 \ \mathrm{m/s^2}\right)$$
$$= 1.3 \times 10^7 \ \mathrm{N}.$$

<div style="text-align:right">11.34</div>

The steel's weight is $m_{\mathrm{w}} g = 9.80 \times 10^7$ N, which is much greater than the buoyant force, so the steel will remain submerged. Note that the buoyant force is rounded to two digits because the density of steel is given to only two digits.

Strategy for (b)

Here we are given the maximum volume of water the steel boat can displace. The buoyant force is the weight of this volume of water.

Solution for (b)

The mass of water displaced is found from its relationship to density and volume, both of which are known. That is,

$$m_{\mathrm{w}} = \rho_{\mathrm{w}} V_{\mathrm{w}} = \left(1.000 \times 10^3 \ \mathrm{kg/m^3}\right)\left(1.00 \times 10^5 \ \mathrm{m^3}\right)$$
$$= 1.00 \times 10^8 \ \mathrm{kg}.$$

<div style="text-align:right">11.35</div>

The maximum buoyant force is the weight of this much water, or

$$F_{\mathrm{B}} = w_{\mathrm{w}} = m_{\mathrm{w}} g = \left(1.00 \times 10^8 \ \mathrm{kg}\right)\left(9.80 \ \mathrm{m/s^2}\right)$$
$$= 9.80 \times 10^8 \ \mathrm{N}.$$

<div style="text-align:right">11.36</div>

Discussion

The maximum buoyant force is ten times the weight of the steel, meaning the ship can carry a load nine times its own weight without sinking.

Making Connections: Take-Home Investigation

A piece of household aluminum foil is 0.016 mm thick. Use a piece of foil that measures 10 cm by 15 cm. (a) What is the mass of this amount of foil? (b) If the foil is folded to give it four sides, and paper clips or washers are added to this "boat," what shape of the boat would allow it to hold the most "cargo" when placed in water? Test your prediction.

Density and Archimedes' Principle

Density plays a crucial role in Archimedes' principle. The average density of an object is what ultimately determines whether it floats. If its average density is less than that of the surrounding fluid, it will float. This is because the fluid, having a higher density, contains more mass and hence more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink.

The extent to which a floating object is submerged depends on how the object's density is related to that of the fluid. In Figure 11.21, for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship loaded. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or

$$\text{fraction submerged} = \frac{V_{\mathrm{sub}}}{V_{\mathrm{obj}}} = \frac{V_{\mathrm{fl}}}{V_{\mathrm{obj}}}.$$

<div style="text-align:right">11.37</div>

The volume submerged equals the volume of fluid displaced, which we call V_{fl}. Now we can obtain the relationship between the densities by substituting $\rho = \frac{m}{V}$ into the expression. This gives

$$\frac{V_{fl}}{V_{obj}} = \frac{m_{fl}/\rho_{fl}}{m_{obj}/\overline{\rho}_{obj}},$$

where $\overline{\rho}_{obj}$ is the average density of the object and ρ_{fl} is the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, and so they cancel from the equation, leaving

$$\text{fraction submerged} = \frac{\overline{\rho}_{obj}}{\rho_{fl}}.$$
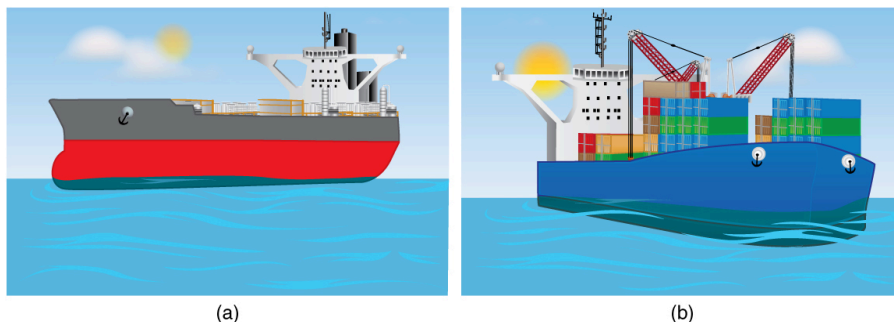
Figure 11.21 An unloaded ship (a) floats higher in the water than a loaded ship (b).

We use this last relationship to measure densities. This is done by measuring the fraction of a floating object that is submerged—for example, with a hydrometer. It is useful to define the ratio of the density of an object to a fluid (usually water) as **specific gravity**:

$$\text{specific gravity} = \frac{\overline{\rho}}{\rho_w},$$

where $\overline{\rho}$ is the average density of the object or substance and ρ_w is the density of water at 4.00°C. Specific gravity is dimensionless, independent of whatever units are used for ρ. If an object floats, its specific gravity is less than one. If it sinks, its specific gravity is greater than one. Moreover, the fraction of a floating object that is submerged equals its specific gravity. If an object's specific gravity is exactly 1, then it will remain suspended in the fluid, neither sinking nor floating. Scuba divers try to obtain this state so that they can hover in the water. We measure the specific gravity of fluids, such as battery acid, radiator fluid, and urine, as an indicator of their condition. One device for measuring specific gravity is shown in Figure 11.22.

> **Specific Gravity**
>
> Specific gravity is the ratio of the density of an object to a fluid (usually water).
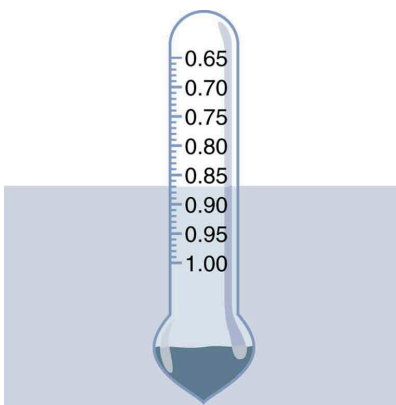
Figure 11.22 This hydrometer is floating in a fluid of specific gravity 0.87. The glass hydrometer is filled with air and weighted with lead at the bottom. It floats highest in the densest fluids and has been calibrated and labeled so that specific gravity can be read from it directly.

✳ EXAMPLE 11.9

Calculating Average Density: Floating Woman

Suppose a 60.0-kg woman floats in freshwater with 97.0% of her volume submerged when her lungs are full of air. What is her average density?

Strategy

We can find the woman's density by solving the equation

$$\text{fraction submerged} = \frac{\overline{\rho}_{\text{obj}}}{\rho_{\text{fl}}}$$

11.41

for the density of the object. This yields

$$\overline{\rho}_{\text{obj}} = \overline{\rho}_{\text{person}} = (\text{fraction submerged}) \cdot \rho_{\text{fl}}.$$

11.42

We know both the fraction submerged and the density of water, and so we can calculate the woman's density.

Solution

Entering the known values into the expression for her density, we obtain

$$\overline{\rho}_{\text{person}} = 0.970 \cdot \left(10^3 \, \frac{\text{kg}}{\text{m}^3} \right) = 970 \frac{\text{kg}}{\text{m}^3}.$$

11.43

Discussion

Her density is less than the fluid density. We expect this because she floats. Body density is one indicator of a person's percent body fat, of interest in medical diagnostics and athletic training. (See Figure 11.23.)
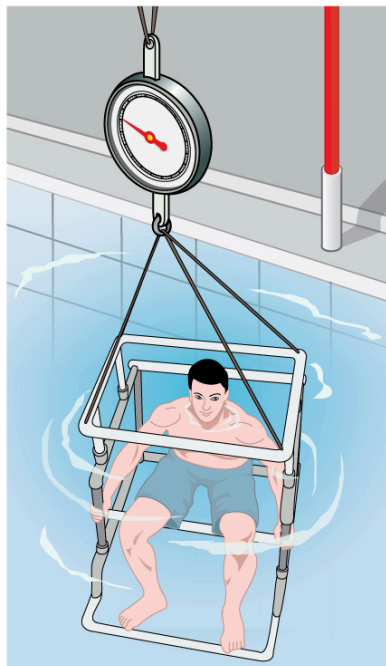

Figure 11.23 Subject in a "fat tank," where he is weighed while completely submerged as part of a body density determination. The subject must completely empty his lungs and hold a metal weight in order to sink. Corrections are made for the residual air in his lungs (measured separately) and the metal weight. His corrected submerged weight, his weight in air, and pinch tests of strategic fatty areas are used to calculate his percent body fat.

There are many obvious examples of lower-density objects or substances floating in higher-density fluids—oil on water, a hot-

air balloon, a bit of cork in wine, an iceberg, and hot wax in a "lava lamp," to name a few. Less obvious examples include lava rising in a volcano and mountain ranges floating on the higher-density crust and mantle beneath them. Even seemingly solid Earth has fluid characteristics.

More Density Measurements

One of the most common techniques for determining density is shown in Figure 11.24.
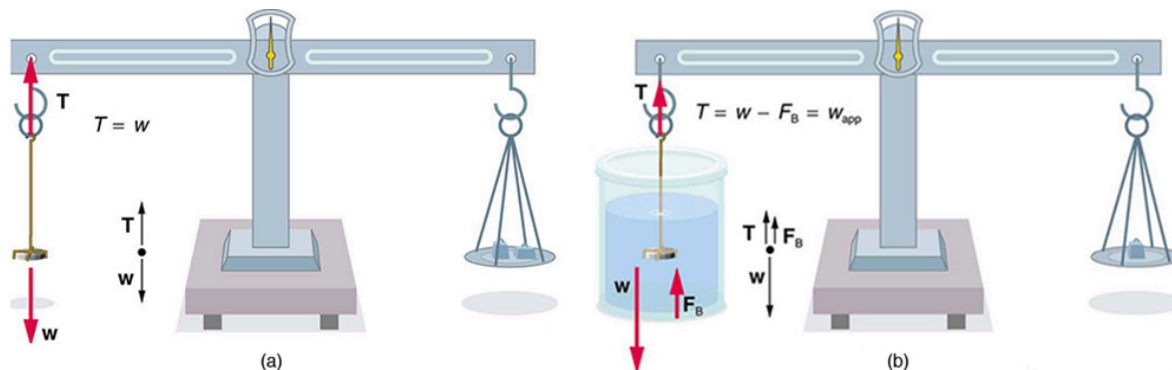
Figure 11.24 (a) A coin is weighed in air. (b) The apparent weight of the coin is determined while it is completely submerged in a fluid of known density. These two measurements are used to calculate the density of the coin.

An object, here a coin, is weighed in air and then weighed again while submerged in a liquid. The density of the coin, an indication of its authenticity, can be calculated if the fluid density is known. This same technique can also be used to determine the density of the fluid if the density of the coin is known. All of these calculations are based on Archimedes' principle.

Archimedes' principle states that the buoyant force on the object equals the weight of the fluid displaced. This, in turn, means that the object *appears* to weigh less when submerged; we call this measurement the object's *apparent weight*. The object suffers an *apparent weight loss* equal to the weight of the fluid displaced. Alternatively, on balances that measure mass, the object suffers an *apparent mass loss* equal to the mass of fluid displaced. That is

$$\text{apparent weight loss} = \text{weight of fluid displaced} \qquad \boxed{11.44}$$

or

$$\text{apparent mass loss} = \text{mass of fluid displaced.} \qquad \boxed{11.45}$$

The next example illustrates the use of this technique.

✳ EXAMPLE 11.10

Calculating Density: Is the Coin Authentic?

The mass of an ancient Greek coin is determined in air to be 8.630 g. When the coin is submerged in water as shown in Figure 11.24, its apparent mass is 7.800 g. Calculate its density, given that water has a density of 1.000 g/cm^3 and that effects caused by the wire suspending the coin are negligible.

Strategy

To calculate the coin's density, we need its mass (which is given) and its volume. The volume of the coin equals the volume of water displaced. The volume of water displaced V_w can be found by solving the equation for density $\rho = \frac{m}{V}$ for V.

Solution

The volume of water is $V_\text{w} = \frac{m_\text{w}}{\rho_\text{w}}$ where m_w is the mass of water displaced. As noted, the mass of the water displaced equals the apparent mass loss, which is $m_\text{w} = 8.630$ g $- 7.800$ g $= 0.830$ g. Thus the volume of water is $V_\text{w} = \frac{0.830 \text{ g}}{1.000 \text{ g/cm}^3} = 0.830$ cm^3. This is also the volume of the coin, since it is completely submerged. We can now find the density of the coin using the definition of density:

$$\rho_c = \frac{m_c}{V_c} = \frac{8.630 \text{ g}}{0.830 \text{ cm}^3} = 10.4 \text{ g/cm}^3.$$

<div style="text-align:right">11.46</div>

Discussion

You can see from Table 11.1 that this density is very close to that of pure silver, appropriate for this type of ancient coin. Most modern counterfeits are not pure silver.

This brings us back to Archimedes' principle and how it came into being. As the story goes, the king of Syracuse gave Archimedes the task of determining whether the royal crown maker was supplying a crown of pure gold. The purity of gold is difficult to determine by color (it can be diluted with other metals and still look as yellow as pure gold), and other analytical techniques had not yet been conceived. Even ancient peoples, however, realized that the density of gold was greater than that of any other then-known substance. Archimedes purportedly agonized over his task and had his inspiration one day while at the public baths, pondering the support the water gave his body. He came up with his now-famous principle, saw how to apply it to determine density, and ran naked down the streets of Syracuse crying "Eureka!" (Greek for "I have found it"). Similar behavior can be observed in contemporary physicists from time to time!

⏩ PHET EXPLORATIONS

Buoyancy

When will objects float and when will they sink? Learn how buoyancy works with blocks. Arrows show the applied forces, and you can modify the properties of the blocks and the fluid.

Click to view content (https://phet.colorado.edu/sims/density-and-buoyancy/buoyancy_en.html)
Figure 11.25

PhET

11.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

Cohesion and Adhesion in Liquids

Children blow soap bubbles and play in the spray of a sprinkler on a hot summer day. (See Figure 11.26.) An underwater spider keeps his air supply in a shiny bubble he carries wrapped around him. A technician draws blood into a small-diameter tube just by touching it to a drop on a pricked finger. A premature infant struggles to inflate her lungs. What is the common thread? All these activities are dominated by the attractive forces between atoms and molecules in liquids—both within a liquid and between the liquid and its surroundings.

Attractive forces between molecules of the same type are called **cohesive forces**. Liquids can, for example, be held in open containers because cohesive forces hold the molecules together. Attractive forces between molecules of different types are called **adhesive forces**. Such forces cause liquid drops to cling to window panes, for example. In this section we examine effects directly attributable to cohesive and adhesive forces in liquids.

> ### Cohesive Forces
>
> Attractive forces between molecules of the same type are called cohesive forces.

> ### Adhesive Forces
>
> Attractive forces between molecules of different types are called adhesive forces.